CHAPTER ONE

Introduction to bevel gears
and
related cutting methods
01.1 - Basic Information

In the world of gears transmission, the bevel gears sector occupies a place apart. While the production of cylindrical gears is, somehow, more easily understandable since the kinematics “tool-gear” is perceivable and can be represented by simple mathematical formulas, the bevel gears subject is more complex and not easy to understand for those who are not expert on this topic. We want to give here an overview of different methods of cutting and the terminology used in dealing with the processing of the bevel gears. Meanwhile, we must say that a pair of bevel gears is formed by a pinion (the wheel with the least number of teeth) and a crown (the other wheel with the most number of teeth): it is used to transmit motion from one shaft to another which has a different direction. Most of the time the angle between the two shafts is 90°, but can also be a different angle. The typical utilization of the bevel gears is as a speed reducer, with the output at 90° to the entry axis (Figure N°01.1)

![Figure N°01.1](image)

This type of transmission was very frequent in the rear axle and in the differential of cars that once had the engine mounted longitudinally. For several years now, in a fairly good part of the cars, the engine is in a transverse position and the traction is on the front axle; so the traction torque to the axle shaft use cylindrical gears. For the cars, the disadvantage of this engine placement is a large turning radius, which in small cars do not have a great importance while, in the more high-end cars, the handling of the steering becomes a critical factor. Some manufacturers have therefore stayed with rear-wheel drive & longitudinal engine and others are now returning to this solution. In addition to the above manufacturers the bevel gears are still commonly used in industrial vehicles, tractors, soil moving machinery and transmissions for many other different applications. In the vast majority of cases, these transmissions use a special type of bevel where the axes do not intersect, but lie on different planes: these are the hypoid gears. These gears have considerable advantages over axial gears: the most important is a greater load capacity at constant volume. Bevel gears can be of different types. In the first place they can be divided into gears with straight teeth and gears with spiral teeth (when the lengthwise profile of tooth is curved: can be an arc of circle or an arc of spiral, epicycloid) (Figure N°01.2).
Bevel gears with spiral teeth result in a smoother action of tooth meshing, with load permanently distributed on two or more teeth and, on the whole, less noisy. They can transmit more torque with the same size.

A variant of spiral bevel gears are the hypoid gears. While the gears represented in Figure N°01.2 have axes which intersect and therefore belong to a same plane, hypoid gears have axes that do not intersect (Figure N°01.3). The transmission of the motion takes place in this case more gradually (greater overlap) and, because the pinion has greater diameter at the same transmission ratio respect a spiral bevel gear, it can then transmit higher powers.

It has however the drawback that there is a considerable slipping between the surfaces of the crown teeth and those of the pinion and then it is often recommended to use lubricants with EP additives (Extreme Pressure).

To give an idea of the complexity of a pair of bevel gears, it is enough consider the amount of parameters that define the teeth.

In Figure N°01.4, are indicated schematically the characteristics of a pair of bevel gears with spur teeth with axes at 90° and, in Table N°01.1 the meaning of the symbols.
Table N°01.1

<table>
<thead>
<tr>
<th>Elements</th>
<th>Pinion</th>
<th>Worm wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure Angle</td>
<td>α</td>
<td></td>
</tr>
<tr>
<td>Module</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Number of teeth</td>
<td>z</td>
<td>Z</td>
</tr>
<tr>
<td>Transmission ratio</td>
<td>φ = \frac{z}{Z}</td>
<td></td>
</tr>
<tr>
<td>Pitch diameter</td>
<td>d_p = z·M</td>
<td>D_p = Z·M</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>α_p1 \left( \tan α_p1 = \frac{z}{r} \right)</td>
<td>α_p2 \left( \tan α_p2 = \frac{z}{r} \right)</td>
</tr>
<tr>
<td>Cone distance</td>
<td>G_p = \frac{d_p}{2 \sin α_{p1}} = \frac{D_p}{2 \sin α_{p2}}</td>
<td></td>
</tr>
<tr>
<td>Back cone distance</td>
<td>r_i = \frac{d_p}{2 \cos α_{p1}} = \frac{r_p}{\cos α_{p1}}</td>
<td>R_i = \frac{R_p}{\cos α_{p2}}</td>
</tr>
<tr>
<td>Number of imaginary teeth</td>
<td>x_i = \frac{z}{\cos α_{p1}}</td>
<td>Z_i = \frac{Z}{\cos α_{p2}}</td>
</tr>
<tr>
<td>Normal Addendum</td>
<td>a_1 = M</td>
<td>a_2 = M</td>
</tr>
<tr>
<td>Normal Dedendum</td>
<td>d_1 = 1,188 M (according Gleason)</td>
<td>d_2 = 1,188 M</td>
</tr>
<tr>
<td>Tooth height</td>
<td>2,188 M</td>
<td></td>
</tr>
<tr>
<td>Normal circular thickness</td>
<td>s_p = \frac{πM}{2}</td>
<td></td>
</tr>
<tr>
<td>Angle addendum</td>
<td>α_{a1} \left( \tan α_{a1} = \frac{a_1}{G_p} \right)</td>
<td>α_{a2} \left( \tan α_{a2} = \frac{a_2}{G_p} \right)</td>
</tr>
<tr>
<td>Angle dedendum</td>
<td>α_{d1} \left( \tan α_{d1} = \frac{d_1}{G_p} \right)</td>
<td>α_{d2} \left( \tan α_{d2} = \frac{d_2}{G_p} \right)</td>
</tr>
<tr>
<td>Outer corner</td>
<td>α_{s1} = α_{p1} + α_{a1}</td>
<td>α_{s2} = α_{p2} + α_{a2}</td>
</tr>
<tr>
<td>Inner corner</td>
<td>α_{i1} = α_{p1} - α_{a1}</td>
<td>α_{i2} = α_{p2} - α_{a2}</td>
</tr>
<tr>
<td>Outside diameter</td>
<td>d_s = d_p + 2α_1 \cos α_{p1}</td>
<td>D_s = D_p + 2α_2 \cos α_{p2}</td>
</tr>
<tr>
<td>Cone distance at face angle</td>
<td>G_s = \frac{G_p}{\cos α_{a1}} = \frac{a_1}{\tan α_{a1}}</td>
<td>G_s = \frac{G_p}{\cos α_{a2}} = \frac{a_2}{\tan α_{a2}}</td>
</tr>
<tr>
<td>Cone distance at root angle</td>
<td>G_t = \frac{G_p}{\cos α_{d1}} = \frac{a_1}{\tan α_{d1}}</td>
<td>G_t = \frac{G_p}{\cos α_{d2}} = \frac{a_2}{\tan α_{d2}}</td>
</tr>
</tbody>
</table>

The nominal module of the gear is what, multiplied by the number of teeth, gives the pitch diameter from the outer side, according with the Gleason geometry, with teeth with double taper. Oerlikon geometry, instead, consider the pitch diameter in the middle of the gear band, because of its geometry with tooth at constant height.

If we then consider a spiral bevel gear there is a further element of complexity related to the helix teething.

The result is that the study of the tool that must generate the conical helical tooth must consider both the geometry of the gear and the relative positioning between gear and tool on the machine.
Figure N°01.8, for example, shows the displacement which must have the mill cutter respect to the center of the gear displacement, which depends not only on the geometry of the tooth but also by the diameter of the cutter.

If you are using the Gleason machines to cut the gears, the data needed to build the milling cutter and the machine settings are reported in "summary tables" included in the provided software.

Bevel gears generally have the tooth bottom with variable width, as well as the thickness and the height of the tooth is variable.

There is a special taper correction called by Gleason the "root tilted angle" or "Duplex Taper", which makes uniform the width of the tooth bottom.

With this operation, the tooth space is wider in the bottom area, thus making it possible to use cutting tools or grinding with more consistent peaks, with obvious advantage on the tool life.
Various types of geometries for bevel gears with spiral teeth

| **Gleason** | ➢ Longitudinal curvature of the tooth: arc (spiral)  
               ➢ Transverse thickness, height of the tooth and tooth space tapered towards the vertex of the cone. Double taper.  
               ➢ Angle of the spiral from 0 ° (toothing Zerol) up to about 45 °; normally about 35 °. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Gleason" /></td>
<td><img src="image" alt="Gleason" /></td>
</tr>
</tbody>
</table>

| **Oerlikon – Spiromatic** | ➢ Longitudinal curvature of the tooth: epicycloid  
                             ➢ Tooth height constant  
                             ➢ Toothing N: the normal module is maximum at the center of the tooth and reduces towards the two sides. Spiral angle typically between 30 ° and 50 °  
                             ➢ Toothing G: the spiral angle from 0 ° to about 50 ° |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Oerlikon – Spiromatic" /></td>
<td><img src="image" alt="Oerlikon – Spiromatic" /></td>
</tr>
</tbody>
</table>

| **Klingelnberg – Palloid** | ➢ Longitudinal curvature of the tooth: involute  
                               ➢ Constant tooth height  
                               ➢ Normal pitch and thickness  
                               ➢ Angle of the spiral typically 35 ° - 38 °  
                               ➢ Slightly faceted surface (caused by the envelope of the conical sections hobs) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Klingelnberg – Palloid" /></td>
<td><img src="image" alt="Klingelnberg – Palloid" /></td>
</tr>
</tbody>
</table>

| **Klingelnberg - Zyclo – Palloid** | ➢ Longitudinal curvature of the tooth: epicycloid  
                                       ➢ Constant height of the teeth  
                                       ➢ Normal module and pitch, according to the angle of the spiral tapered down to almost constant  
                                       ➢ Angles of the spiral from 0 ° to about 45° |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Klingelnberg - Zyclo – Palloid" /></td>
<td><img src="image" alt="Klingelnberg - Zyclo – Palloid" /></td>
</tr>
</tbody>
</table>

| **Modul – Kurvex** | ➢ Longitudinal curvature of the tooth: arc of circle  
                      ➢ Tooth height constant or tapered  
                      ➢ Corners of the spiral 25° to about 45° |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Modul – Kurvex" /></td>
<td><img src="image" alt="Modul – Kurvex" /></td>
</tr>
</tbody>
</table>
01.2 – Production methods of bevel gears

01.2.1 Some classifications of bevel gears:

**Angle between axes**

Angle between the axes = 90°

Angle between the axes smaller than 90°

Angle between the axes greater than 90°

**Tooth height**

Constant tooth height

Not constant tooth height

Figure N°01.7

Figure N°01.8
Longitudinal curvature of the tooth

**Arc of circle**

**Epicycloid**

**Involute**

Figure N°01.9

**System index: that is succession of teeth spaces cut**

Continuous method: all the spaces are cut together (Face Hobbing)

Discontinuous method: the spaces are cut one at a time (Face Milling)

Figure N°01.10
Cutting method

Pinion and crown "generate"

Pinion generate and crown “formate”
(that is, working with a form milling cutter)

Figure N°01.11

Table of comparison between Face Milling and Face Hobbing methods

<table>
<thead>
<tr>
<th>Face Milling or single indexing</th>
<th>Face Hobbing o continuous indexing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread blade (or fixed setting or five cut)</td>
<td>Completing method</td>
</tr>
<tr>
<td>2 cutting operations for the crown</td>
<td></td>
</tr>
<tr>
<td>Roughing</td>
<td>1 cutting operation for part</td>
</tr>
<tr>
<td>Finish</td>
<td></td>
</tr>
<tr>
<td>3 cutting operations for the pinion</td>
<td>It is sufficient one machine</td>
</tr>
<tr>
<td>Roughing</td>
<td>Less operations of loading / unloading</td>
</tr>
<tr>
<td>Finish the release side (concave side)</td>
<td>Less need for space</td>
</tr>
<tr>
<td>Finish the pull side (convex side)</td>
<td>Increased flexibility in production</td>
</tr>
</tbody>
</table>

**Duplex:**
Completing methods for pinion and crown
It is sufficient one machine
Less operations of loading / unloading
Less need for space
Increased flexibility in production
Each space is subsequently cut
Longitudinal curvature:
Arc (constant radius)

All spaces are cut simultaneously with the previous
Elongated Epicycloid:
(non-constant radius)

The rays of the outer blade (OB) and the inner blade (IB) are different

The rays of the outer blade (OB) and the inner blade (IB) are theoretically identical
The height of the tooth is tapered
The topland is constant

Tooth height is constant
The topland decreases from heel to toe

The generating crown gears are different between the pinion and crown.
The wheel axles do not coincide:
  tooth is not constant

The generating crown gears are virtually identical between the pinion and crown.
The wheel axles coincide:
  constant tooth height.
The cutting lines are parallel (coincide) with the contact lines.
The cutting lines intersect the contact lines with a corner.
The cutting lines are shorter than with Face Hobbing. With remarkable feeds the cutting lines could appear as flats.

The marks of advancement (generated by the cutting lines) are parallel to the contact lines.
The marks of advancement (generated by the cutting lines) are not parallel to the contact lines.
The sharpening and the positioning of the blades is critical for a good rolling without noise generation. You can use wider blades position tolerances without rolling problems.
Cutting with the method Face Milling

The Face Milling is a method of discontinuous cutting, which means cutting completely a tooth compartment and then switching to the next tooth space.

![Figure N°01.18](image)

The mill cutter is positioned offset respect to the center of the workpiece of the values O'A and O'B, so that the average diameter of the cutter is tangent to the tooth side, which mean an inclination of the angle β in the middle point of the tooth (Figure N° 1.19). It is noted that the nominal spiral angle is the one present in mid of the toothed strap for bevel gears spirals; in reality are cut arcs of circumference that are particular types of spiral.

![Figure N°01.19](image)
Cutting with the method Face Hobbing

The other method used for the production of spiral bevel gears is the “Face Hobbing”, which is a method of continuous cutting. The work-piece rotates in sync with the tool so that the cutter teeth mesh with the teeth of the work-piece, the same way as happens with the teeth of cylindrical gears with the hob. Figure N°1.20 represent schematically this system.

![Figure N°1.20](image1)

Figure N°01.20 – Cutting method Face Hobbing

While the piece rotates in the direction V, the tool, after having cut a space with a pair of cutting edges, enters with another pair of cutting edges (1) in the following space (A). The cutting edges of the cutter, therefore, are not arranged on a circumference coincident with the center of rotation, but on spirals, as is clearly seen in Figure N°01.21 which represents a cutter Gleason with blades type Pentac FH® for the cutting method Face Hobbing.

![Figure N°01.21](image2)

Figure N°01.21 – Gleason mill cutter for Face Hobbing with Pentac® blades
The processing with Face Milling system includes two methods:

- “Formate”
- “Generate”

With the **Formate method** you are working with a form milling cutter, which is used for crown toothing. Possible errors on a blade does not influence the quality of the generated surface, because on each compartment pass all the teeth of the milling cutter which all have the shape of the compartment to be generated. The method, very productive, applies to pairs of spiral bevel gears with ratio greater than 1:2.5.

With the **Generate method** works for generation (similarly to what happens with the hob for worm wheels). The teeth of the cutter represent a generating crown gear slot that, rolling with the pinion or crown, generates the tooth spaces. Eventual errors on a blade cause an error on the side of the gear tooth, because each blade generates a piece of tooth profile, and then, in the event of error on the blade would have an error on a section of the tooth profile.

01.2.1 **Bevel gears with spur teeth**

The bevel gears are the simplest and still the most widely used. They have a linear contact footprint along the cone distance of the tooth; Gleason has introduced the method "Coniflex" which generates a longitudinal camber: this allows better insensitivity to the misalignment.

For the wheels for differential (the most common), since normally do not run but only transmit torque, Gleason proposes a method of cutting "Revacycle" very productive, being essentially a broaching. However, it is also gaining ground the molding in net shape which allows to obtain the finished surface of the teeth of the mold.

Cutting methods: always a tooth at a time, face milling type, machines Coniflex Gleason 102, 104, 114, or with Modul machines. For high productivity machine Gleason Revacycle.

Most updated methods: Gleason Coniflex plus, basically Coniflex on CNC machine, but with carbide stick blade, with different placements of a single cutter with radial toothing (in place of the pair of blades Coniflex, made with carbide stick blades, or Oerlikon Hypoflex, made with circular cutters composed of carbide stick blades by interpolation of axes of the CNC machine.

Before we look at the types of tools used for cutting bevel gears, we must point out that the positioning of the milling cutter, with the method shown in Figure N°1.19 must be supplemented by a proper axial positioning of the workpiece relative to the cutter. Any set-up error leads to an error in the profile or on the spiral. The errors of this type cause an incorrect contact between the surfaces of the teeth of the worm wheel and the pinion.

But we must consider that the optimum contact, which is what ensures the lowest noise and maximum efficiency of the coupling must take place under load, with thermally treated gears. The teeth of the gears under load will deform, exactly as happens in the cylindrical gears, varying the contact area.

To have a right contact under load is therefore necessary to perform the gears with profiles and spirals which compensate the deformation under load. The teeth of the gears under load will deform, exactly as happens in the cylindrical gears, varying the contact area.
To have a right contact under load is therefore necessary to perform the gears with profiles and spirals which compensate the deformation under load.

There are some very sophisticated software that, according to the load and the characteristics of the bevel gear pair, determine in advance such compensation but, more frequently, a check is done with a tester that works under weak load and examine the trace of the contact, drawing the corrections be applied to restore the contacts in the best area.

Depending on the type of contact you make the necessary corrections of the set up, very easy today on CNC machines.

The Table N°01.2 shows the possible contact types and the type of adjustment to make. If the gear is not subject to finishing operations after the heat treatment, for example grinding of the teeth or re-machining with carbide cutters, one must consider the deformations due to the heat treatment and, also in this case, will have to be modified the profiles and the spirals obtained in the process of toothing in order to per-correct these deformation.

**Table N°01.2**

<table>
<thead>
<tr>
<th>Concave side</th>
<th>Convex side</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td>Contact OK</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td>1) Move the worm wheel or the pinion at the summit</td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td>2) Move the worm wheel and the pinion from the top</td>
</tr>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td>3) Increase the vertical displacement</td>
</tr>
<tr>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td>4) Decrease the vertical displacement</td>
</tr>
<tr>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td>5) Actions 2 and 3 combined</td>
</tr>
<tr>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td>6) Actions 2 and 3 combined</td>
</tr>
</tbody>
</table>
Choice of milling cutter diameter

The average diameter of the milling cutter is chosen according to the characteristics of the teeth to be realized and in particular we must consider the module, the height of the tooth to be performed, the maximum face width.

The average diameter of the milling cutter is not strictly binding, in the sense that has some variation possibilities.

Table N°01.3 shows an indication the diameters of the mill cutter according to the characteristics of the gears.

<table>
<thead>
<tr>
<th>Average diameter of the milling cutter</th>
<th>1 1/16”</th>
<th>1 ½”</th>
<th>2”</th>
<th>3 ½”</th>
<th>6”</th>
<th>7 ½”</th>
<th>9”</th>
<th>12”</th>
<th>18”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max module (mm)</td>
<td>1,75</td>
<td>2,5</td>
<td>2,5</td>
<td>4,5</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Max tooth height (mm)</td>
<td>3,5</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>10</td>
<td>14</td>
<td>14</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Max face width (mm)</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>20</td>
<td>32</td>
<td>38</td>
<td>45</td>
<td>65</td>
<td>100</td>
</tr>
</tbody>
</table>
01.3 - Summary of the main production systems of bevel gears
01.4 - Finishing of the bevel gears

The milling operation of conical teeth, is followed by heat treatments that serve to enhance the characteristics of the various steels used to build the gears. As for almost all the mechanical parts subject to alternating loads and / or contact with other bodies with which they exchange efforts, there are two schools of thought for the thermal treatment / choice of building materials: one is the American school which chooses materials with high content of carbon, low alloy, for surface hardening; while the European school chooses high alloy materials, with low carbon, suitable for carburizing. In the two cases the sizing is different because there are different fatigue limits of the two families of materials.

In any case the heat treatment, which determines phase transitions of the materials, deform the components; then after the heat treatment, very often the pinions (for their "mushroom shape") and sometimes even the crowns, are straightened; the diameters of the bearing seat are brought, by grinding or turning, to the size that the designer has determined for the correct positioning of the rolling elements.

01.4.1 Lapping finishing

Both the bevel gears obtained with Face Hobbing and those obtained with Milling Face are lapped. This operation, obtained by special machines (called lapping machines), provides pinion and gear to roll together under a moderate torque. A lapping compound (oil and abrasive) is pumped between the meshing teeth that roll in various mutual positions: in this way, a few hundredths of a surface material are removed in areas previously determined. The two pieces compose now a pair no longer interchangeable, since the surfaces are adapted to each other, thus compensating the distortions induced by the heat treatment. A further verification done on a tester machine determines the relative position of the pinion and the crown, where transmission errors are minimal: that position must be also respected in the assembly to have less rolling noise.

01.4.2 Grinding finishing

Very modern CNC grinding machines are able to grind the surfaces of the tooth spaces only in the case of Face Milling; surfaces are verified with three-dimensional measuring machines, opportune adapted to explore the topography of the tooth. The grinding machines are able to reproduce and maintain topographies of few microns and so the thickness of the tooth. At this point it is sufficient a statistical measurement of the teeth topography, and then the two productions of pinion and crown are moving forward in parallel to arrive at the assembly where, randomly, can be mounted without verification of the installation location.

01.4.3 Hard skiving finishing

In the case of Face Hobbing is possible to recover the heat treatment deformation through the hard skiving, that is a cutter that mounts carbide tools. With this cutter is possible to cut a depth of 0.1 mm and remove such distortions. This method is particularly delicate and it applies to large pairs that can not be rectified. The above is just a simple information at completion of the discussion on the bevel gears. Of course, it is not exhaustive nor comprehensive of all the problems related to the production of bevel gears.